Follow the Bouncing Ball
MCIS
Mathematicians Can Investigate Stuff

0. In 1965, the Super Ball made its debut in toy stores. Its claim to fame was that it bounced better than other balls, a simple idea that sold millions.
   You can watch a couple of commercials and get some history on this craze at:
   http://www.wham-o.com/brands/superball.html
   If you do watch the commercials, *NOTE* you will not get a helicopter to use during this lab.

   Many things that were once popular experience a comeback. Perhaps there is an opportunity here for some entrepreneur, interested in making money? Also, the quality of a ball's bounce is often a key selling point for many different types of balls such as: basket balls, tennis, balls, golf balls, bowling balls… OK, not bowling balls.

1. Since we have no crimes or court cases to investigate right now, let's make use of your team's quantitative talents to become experts at the bounce of a ball. Maybe Wham-O will hire us as consultants?

   Before beginning experimentation, your teams should do some "brain storming". Identify attributes of a ball and bouncing a ball that you can observe or measure. Sort these factors into two categories, those that are related to the bounce and those that are not related to the bounce. Write your list below.

2. From your list of items identify at least two that you suspect may have a functional relationship and identify which you consider the independent and which you consider the dependent variable in that functional relationship.

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Part A

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Rebound

1. Let's call the maximum height a ball reaches on the first bounce, after it has been dropped, the "rebound height". Since you will initially have only one ball to experiment with, which of the factors you have identified do you think will have an effect on the rebound height? Sketch a graph to illustrate your prediction of the functional relationship. Be sure to label the axis on your sketch to reflect the independent and dependent variable.

2. Get a meter stick and a ball for testing. Perform experiments to get a good variety of data for analysis. Organize and record your data in a table below. Be sure everyone on your team also has the data stored in list on their calculator and can generate a scatter plot of the information.

3. Does there appear to be a pattern to the data? How does the scatter plot compare to the sketch you made in (1) above? Based on your scatter plot what functional model do you think will best represent the relationship between the two variables you are experimenting with? Explain your reasoning.

4. Use your calculator to generate your chosen functional model for the data and record the result here.

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5. What is the r^2 for your model and what does it tell you about how well your model represents the data you collected?

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6. Based on your model for your ball, what would the rebound height be if the ball were dropped from a height of 3 yards? Show work.

7. How high would you have to drop the ball from to have it rebound to a height of 40 cm? Show work.

8. If it were ten degrees hotter, do you think your function would need to be modified? Explain your reasoning.

9. If we could change only the color of the ball, do you think your function would need to be modified? Explain your reasoning.

10. Compare your team's mathematical model to that of all groups in the class and establish a ranking of all the balls being tested from best bouncer to worst based on the functions established by each group. Explain the reasoning behind saying one ball bounces better than another.

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Part B
One Good Bounce

1. During your experiments in **Part A**, you only collected one value during a bounce of the ball, the rebound height. Time for a more in depth analysis of a bounce.

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To help with this you will need to get the CBL 2, and the ultrasonic motion detector. The goal now is to capture data recording at least 40 height measurements during one bounce. Use any program or application you wish to set up a data collection experiment to accomplish the goal.

2. Before actually running the experiment lets discuss some challenges. In the Beach Ball Toss lab you had the motion detector on the ground and used a large ball as the target to reflect a signal for the detector. You did not get data points when the ball was very close to the ground. Now you have a smaller target and we definitely want data points down to the ground. To accomplish this you will need to hold the motion detector above the ball, keeping it at a constant height above the floor and over the ball during the data collection so that you get a signal to and from the ball not the floor. This is not easy and may take a few tries to get what you need. Be careful to hold the motion detector level and try to avoid getting a signal bouncing off something other than your intended target, the ball.

3. The next challenge you face is making the data represent the ball's distance from the ground not its distance from the motion detector.
   i. If you were holding the motion detector 5 feet above the ground and a data point shows the ball is 2 feet from the motion detector, how far would the ball be from the ground?

   ii. With the motion detector still 5 feet above the ground, what would the distance from the motion detector be when the ball hits the ground?

   iii. If you were holding the motion detector at a height of 6 feet above the ground and a data point indicated the ball was 5 feet from the motion detector, how far would the ball have been above the ground?

   iv. Explain your plan for processing all the data you collected for (distances from the motion detector) into (heights of the ball from the ground).

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4. Carry out you plan now and make a data set for heights of the ball above the ground. Use this data to make a scatterplot showing height of the ball above the ground during your one complete bounce. This means you will have to get rid of data points that occurred before the bounce started and data point that were collected after the bounce ends.

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5. Use your calculator to generate the mathematical function that best represents your scatterplot. Record the function below. Also, give definitions for both variables (including units of measure) and the appropriate domain and range for the function.

6. Based on your mathematical model for this bounce:

   i. How long was the ball in the air during the bounce?

   ii. What was the maximum height of the ball during the bounce?

   iii. When was the ball at a height of 6 inches?

   iv. What color was the ball?

   v. What was the height of the ball .33 seconds into this bounce?

   vi. When was the ball travelling the fastest? When was it the slowest? Explain your reasoning.

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Part C

Bounce Bounce Bounce Bounce

1. Now it is time for the ultimate challenge. Design and carry out an experiment to capture a succession of bounce after the ball is dropped. Let's make the goal to get at least four complete bounces for one experiment. You will want to process the distance values as you did in **Part B** so that you have a list containing heights

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above the ground instead of distances from the motion detector. This process might be described by the following formula:

Height of ball = Height of motion detector – Balls distance from motion detector

Or using function notation something like:

$$H(t) = H_m - D(t)$$

Where:

t = the time since of data collection (seconds)
$H(t)$ = the height of the ball at time t (the units you choose)
H_m = the height of the motion detector above the ground (the units you choose)
$D(t)$ = the distance of the ball from the motion detector at time t (the units you choose)

2. Once you have captured good data and converted distance from the motion detector into heights above the ground make a scatter plot on your calculator showing height of the ball as a function of time and be sure you are showing data points for at least four complete bounces. Sketch a graph of your scatter plot.

3. Explain what happens from one bounce to the next. How does your work in **Part A** account for this?

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4. From your data about heights above the ground estimate the maximum height of the ball for each of the four (or more) bounces during your experiment. Record these values in the table below:

Bounce	Maximum height
1	
2	
3	

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4	
5	

You might consider including a Bounce value of zero in your data set. What maximum height value would you use for Bounce zero?

5. Determine a mathematical function that best fits the data points you have collected in (4.) Record the model below. Graph, on your calculator, both a scatter plot of the data points and your function. Be sure everyone on your team has these graphs on their calculator.

6. What does your function predict the maximum height would be on the sixth bounce? Show work.

7. How many bounces would you predict before the maximum height falls below one inch? Explain your reasoning.

8. Compare the function you have just determined for your ball to that of the rest of the teams in the class. Explain how looking at this kind of function can be useful in comparing the bouncing qualities of different balls.

9. During this lab, in your investigations of ball bouncing, list the types of mathematical function that you have used to model data.

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